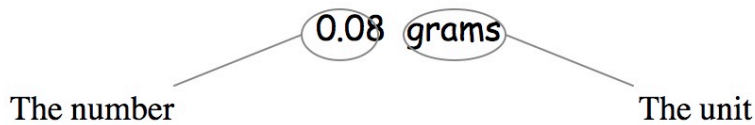


The Unit Factor Method for Physiology Questions

Many laboratory questions in this physiology course will require you to perform a mathematical calculation. For example, “Dr. Gomez orders 0.08 grams of medication for a patient. Each pill contains 0.003 grams of the medicine. How many pills will the patient need?” This problem may seem difficult at first but there is a problem-solving method called the **unit factor method** that will guide you, step by step, to the correct answer. This method is sometimes also called the “dimensional analysis” method.

A) Numbers and units

Any value or quantity in a problem has two parts: A number and a unit. For example,



The unit tells you what *type of thing* you are talking about. The number tells you how much of that thing you are talking about.

B) Making unit conversion factors

What is a unit conversion factor? A unit conversion factor is anything that relates one quantity to another quantity. For example, the phrase “60 seconds per minute” is a unit conversion factor because it relates seconds to minutes. “Peaches are \$1.50 a pound” is a unit conversion factor that relates pounds of peaches to dollars. “Four legs per dog” is a unit conversion factor that relates legs to dogs. Unit conversion factors are important because they are what you use to solve medication problems, like the Dr. Gomez medication problem given at the start of this handout.

All unit conversion factors can be written as a fraction, with one quantity on top of the fraction and the other quantity on the bottom of the fraction. For example, the unit conversion factor “60 seconds per minute” can be written as:

$$\frac{60 \text{ seconds}}{1 \text{ minute}}$$

Notice that the word “per” tells you what goes above and what goes below the fraction line. Whatever comes before the word “per” always goes above the fraction line and whatever comes after the word “per” always goes below the fraction line. You can think of the word “per” as meaning “a fraction line” or “divided by.” In fact, the unit conversion factor written above is read aloud by saying “60 seconds **per** minute.”

Also notice the number 1 was put before the minute unit. Each unit must always have a number in front of it. If the phrase doesn't tell you the number for a unit, assume the number is one.

The word "equals" works the same way as the word "per." For example, you know the phrase "12 inches equals one foot." From this you can make a conversion factor:

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

Just like the word "per," the word "equals" means you should draw a fraction line. What comes before the word "equals" always goes above the fraction line and what comes after the word "equals" always goes below the fraction line.

Often the unit conversion factor you need to solve a problem is not given to you directly in the problem. So where does it come from? From you. You can make a unit conversion factor from any phrase you know that contains the word "per" or the word "equals." You can even make unit conversion factors out of phrases that don't contain the words "per" or "equals." Any phrase that relates one quantity to another is a unit conversion factor. Some examples are given below:

Gasoline costs \$1.90 a gallon.
("1.90 dollars per gallon")

$$\frac{1.90 \text{ dollars}}{1 \text{ gallon}}$$

Nickels weigh 4.5 grams each.
("4.5 grams per nickel")

$$\frac{4.5 \text{ grams}}{1 \text{ nickel}}$$

Every day the cat eats 3 cups of catfood.
("3 cups catfood per day")

$$\frac{3 \text{ cups catfood}}{1 \text{ day}}$$

When you look at a written unit conversion factor, whatever value is on top of the fraction line is called the numerator. Whatever value is below the fraction line is called the denominator. In the previous example, **3 cups catfood** is the numerator and **1 day** is the denominator.

The last thing you need to know about unit conversion factors is that you are always free to flip them upside down whenever you need to. This is called inverting them.

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

can become...

$$\frac{1 \text{ foot}}{12 \text{ inches}}$$

So unit conversion factors always come in pairs. Each member of the pair is made by inverting the numerator and denominator of the other.

Exercise 1: For each unit below, write a pair of unit conversion factors that relate it to some other unit. The first one is done for you as an example. If you get stuck, ask your instructor for help.

a) Days $\frac{7 \text{ days}}{1 \text{ week}}$ and $\frac{1 \text{ week}}{7 \text{ days}}$ b) Pennies

c) Yards d) Quarts

Show your instructor your answers to this exercise before moving on to the next section.

C) An example of using the unit factor conversion method

As an example of how to use the unit conversion factor method to solve a problem, we will use the following problem: “An experiment is done to see the health effect of large amounts of aspirin on rats. Each rat gets 22 grams of aspirin. If 60 rats are used in the experiment, how many grams of aspirin will be used?”

Step 1: Write the answer unit Read the problem and find what units the answer must have. The answer units are usually the units after the words “how many” or “how much.”
In

= _____ grams aspirin

our example, the problem asks “how many grams of aspirin” so grams of aspirin are the answer unit. Write “= _____ grams aspirin” on the right side of the paper:

Step 2: Find a unit conversion factor that involves the answer unit Read the problem and find a sentence that relates the answer unit to something else. In our example, one sentence reads “Each rat gets 22 grams of aspirin.” This sentence relates the answer unit (grams of aspirin) to something else (rats).

Step 3: Write a unit conversion factor based on the sentence. The unit conversion factor must have the *answer units on the top*. Remember that sometimes the problem won't state the unit conversion factor if it is something that most people already know (such as "60 minutes to an hour").

$$\frac{22 \text{ grams aspirin}}{1 \text{ rat}} = \underline{\hspace{2cm}} \text{ grams aspirin}$$

Step 4: Find a value in the problem that has the same units that appear on the bottom of the unit conversion factor. In our problem, the units that appear on the bottom of the unit conversion factor are "rat," so look for a value in the problem that relates to rats. "If 60 rats are used in the experiment..." is the value statement relating to rats.

$$60 \text{ rats} \quad \frac{22 \text{ grams aspirin}}{1 \text{ rat}} = \underline{\hspace{2cm}} \text{ grams aspirin}$$

Step 5: Multiply and check to see that the units cancel. This means that for each unit on the bottom of a fraction, you must find the same unit at the top of a fraction. Units that are not shown as fractions (like "60 rats" in this problem) are considered to be at the top of a fraction. You can check that the units cancel by writing a slash through each unit that appears at the bottom and at the top of a fraction. In our example, the **rat** units cancel.

$$60 \cancel{\text{ rats}} \times \frac{22 \text{ grams aspirin}}{1 \cancel{\text{ rat}}} = \underline{\hspace{2cm}} \text{ grams aspirin}$$

If you have done this step correctly, the only unit that does not cancel is the answer unit (which is **grams aspirin** in this problem).

Step 6: Carry out the multiplication. This is the step where you use your calculator. You should multiply together all the numbers on top of the fractions and divide that answer by all the numbers on the bottom of the fractions. The buttons you should press on your calculator for this problem are:

The correct answer should be 1320. You may have noticed that the last buttons you pushed (dividing by 1) did not change your answer. Dividing by 1 never changes a number. Nevertheless, it's good to get in the habit of dividing by all denominator numbers because in other problems it may not be 1 that you are dividing by.

Exercise 2: Use the unit factor method to solve the following problems. Set up each problem **in the exact same way (using all six steps) as the example problem on the previous pages**. Show all your work. If you get stuck, ask your instructor for help.

- a) Apples sell for 43 cents a pound. How much do 3.2 pounds of apples cost?

- b) Gold costs \$500 per ounce. A solid gold nugget is 23.5 ounces. How much is it worth?

- c) The new hybrid cars get 65 miles per gallon. How many miles can you drive on 5 gallons of gas?

- d) How many days are there in two and a half years?

- e) Dr. Gomez orders 0.08 grams of medication for a patient. Each pill contains 0.003 grams of the medicine. How many pills will the patient need? (Note: When dispensing medicine, if the calculation answer contains a fraction of a pill (such as 52.6 pills) health workers usually round down so as to avoid any chance of overdosing the patient).

Show your instructor your answers to this exercise before moving on to the next section.

D) Problems that require more than one unit conversion factor

The problems you have worked so far can be solved using one single unit conversion factor. Some problems require two unit conversion factors to solve. Here is an example of one of these problems:

How many seconds are there in 5 hours?

There is no common phrase that relates seconds per hour, so you can't solve the problem with a single unit conversion factor. It will require two unit conversion factors. Here is how the problem is set up:

$$5 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = \underline{\hspace{2cm}} \text{ seconds}$$

First of all, notice that the problem is set up in the exact same way as a single unit conversion factor problem: Starting units (hours) on the left, answer units (seconds) on the right, and conversion factors between them.

And as before, all the units (except the answer units “seconds”) must cancel out.

How do you pick the right two unit conversion factors? For the first one, pick a unit conversion factor that will cancel out the starting units. In other words, your first unit conversion factor must have the starting units under the fraction line. The second unit conversion factor must have the answer units on top. On the bottom it must have the top units *from the first conversion factor*.

To put it simply, each unit conversion factor must cancel out the top units to its left.

Exercise 3: Use the unit factor method to solve the following problems. Show all your work, including the unit factors and the cancelled units. If you get stuck, ask your instructor for help. Show your instructor your answers to this exercise before turning it in.

- a) How many inches are there in 110 yards?

- b) Sarah can study 20 pages per night. Each page has three homework problems. If she studies for 3 nights how many homework problems will she do?

- c) Dr. Michael prescribes 300 mg a day of medicine to a patient. Each pill has 15 mg of medicine. How many pills will the patient need for 7 days?

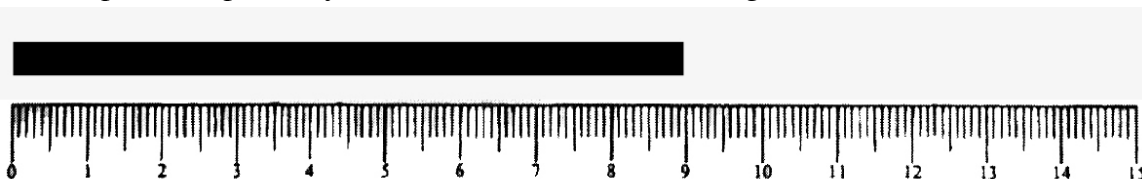
- d) A solution contains 5 grams of glucose per 100 milliliters. Each mole of glucose weighs 180 grams. How many moles are there in 200 milliliters of the glucose solution?

E) Instrument and graph graduations

Very often in the medical field instruments are used to measure something about the patient. For example, blood pressure cuffs measure the patient's blood pressure, thermometers measure their temperature, rulers their height, and spirometers measure the volume of air the patient is breathing. Medical instruments almost always have lines called **graduations** that are used to read the measurement. For example, the centimeter (cm) ruler below has 150 graduations. Notice that a few of the graduations have numbers but most of the graduations do not have numbers.



If you had to measure the black shape below, it is easy to see that it is 9 cm because the shape lines up exactly with the 0 and the 9 numbered graduations.



But often the thing you are measuring falls on one of the unnumbered graduations, as is shown below:



By inspecting the illustration closely, you will see the black shape is 4 graduations past 9 cm. To measure the length correctly, you need to make a unit conversion factor.

Making a unit conversion factor from graduations:

- 1) Pick any two numbered graduations on the instrument.
- 2) Find the difference between the two numbers by subtracting one from the other.
- 3) Divide this difference by the number of graduations between the numbers*.

$$\text{Unit conversion factor for graduations} = \frac{\text{Difference between the two numbers}}{\text{Graduations between the two numbers}}$$

* Include the graduation of the final number as one of the graduations.

We will demonstrate this method using the ruler below.

- 1) Let's choose 0 cm and 1 cm as the two numbered graduations on the ruler.
- 2) The difference is obtained by subtraction: 1 cm – 0 cm = 1 cm.
- 3) There are 10 graduations between the 0 and the 1, so the unit conversion factor is:

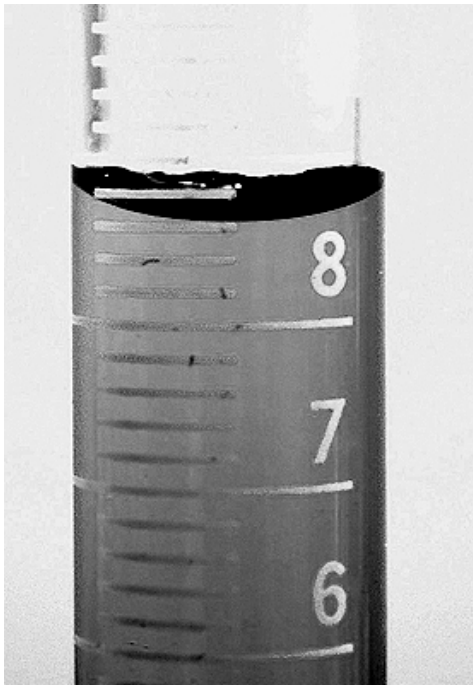
$$\text{Unit conversion factor} = \frac{1 \text{ cm}}{10 \text{ graduations}}$$



Now that we have obtained the value of each graduation, we can calculate the length of the black shape using the unit factor method:

$$\text{Length} = 9 \text{ cm} + (4 \text{ graduations}) \times (1 \text{ cm} / 10 \text{ graduations}) = 9.4 \text{ cm}.$$

Here is another example, this time using a graduated cylinder to measure the volume of a liquid. The meniscus (the bottom of the liquid's curved surface) is 3 graduations above 8 milliliters (ml). To find the volume of the liquid, we first need to calculate the value of each graduation.



1) Let's choose 8 ml and 7 ml as the two numbered graduations.

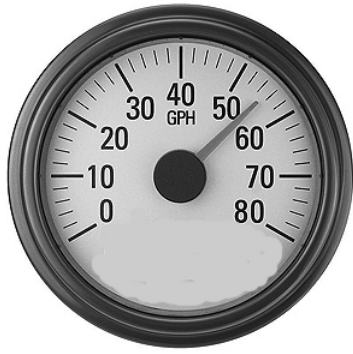
2) The difference between them is 1 ml.

3) There are 5 graduations between them, so the unit conversion factor is 1 ml / 5 graduations

The total volume of the liquid is therefore

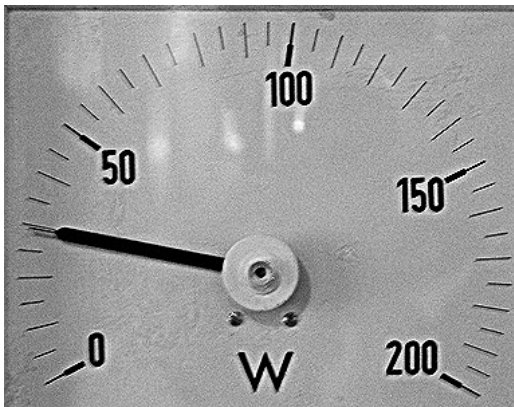
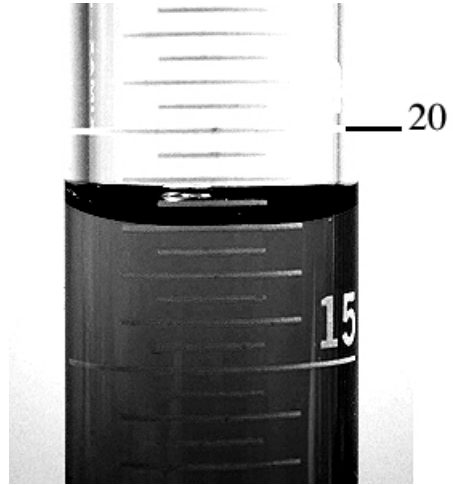
$$8 \text{ ml} + (3 \text{ graduations}) \times (1 \text{ ml} / 5 \text{ graduations}) = 8.6 \text{ ml}.$$

Correctly read the instruments on the next page using the unit conversion factor method presented in this section. Show all your calculations.



= _____ GPH

ml = _____



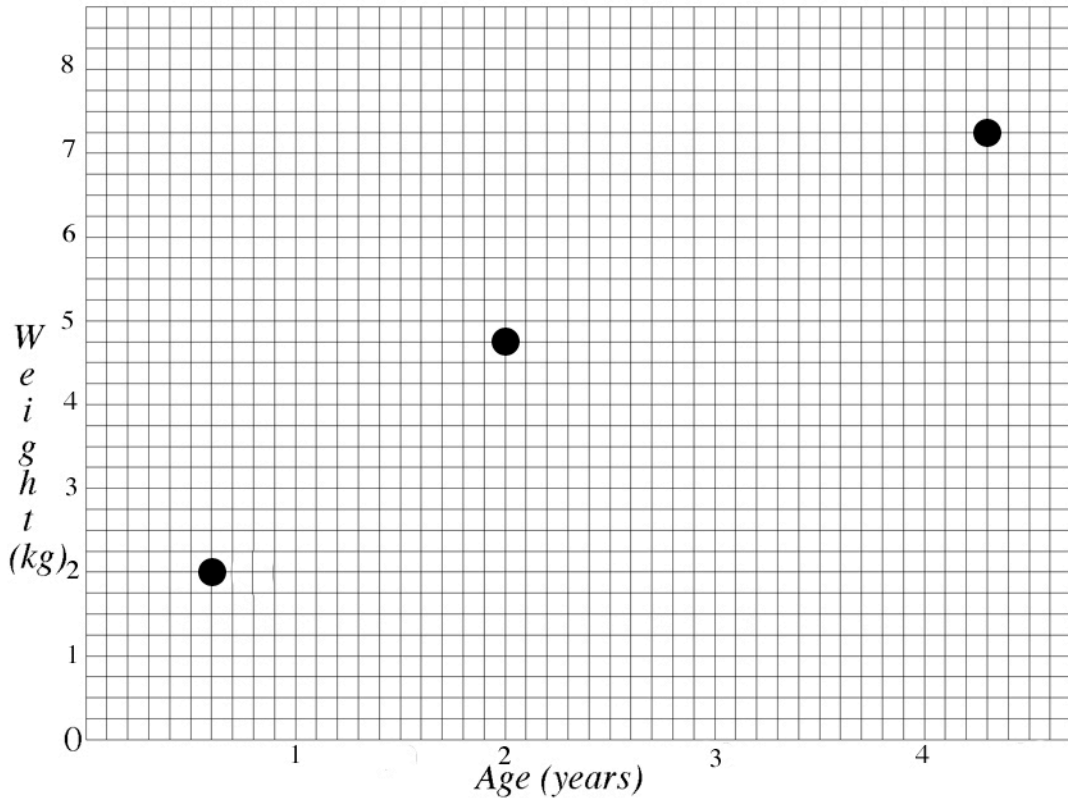
= _____ Watts

ml = _____



Up to this point, we have used medical instruments as examples of correctly reading graduations. The same method can also be used to correctly read the graduations on graphs.

Inspect the graph below of weight in kilograms (kg) versus age for a laboratory rabbit. Then answer the questions below the graph. Show all your calculations.



a) Write the unit conversion factor for the Y-axis (the weight axis): In other words, write a unit conversion factor that relates kilograms to Y-axis lines.

b) Write the unit conversion factor for the X-axis (the age axis): In other words, write a unit conversion factor that relates years to X-axis lines.

Now use these unit conversion factors (or their reciprocals) to answer these questions:

c) On the first data point, how old was the rabbit? _____. On the second data point, what did the rabbit weigh? _____. On the third data point, the rabbit's age was _____ and its weight was _____.

d) How many lines on the Y-axis is 7.4 kg? _____. How many lines on the X-axis is 3.75 years? _____.

e) Draw one dot on the graph that represents a rabbit of 3.4 years and 5.75 kg.

F) Some on-line resources for practicing the Unit Conversion Factor method

Several websites offer practice problems for using the unit conversion factor method to solve problems. A good starting point to find these websites is to search the internet using these keywords: Dimensional analysis practice questions.

I have listed a few of these websites below. These four websites have introductory instruction on unit conversion factors (dimensional analysis).

<http://www.alysion.org/dimensional/fun.htm>

<http://www.purplemath.com/modules/units.htm>

<http://www.chem.tamu.edu/class/fyp/mathrev/mr-da.html>

<http://www.wfu.edu/~ylwong/chem/dimensionanalysis/example1.html>

This website has a dimensional analysis quiz you can take:

http://www2.selu.edu/Academics/Faculty/wparkinson/help/dimensional_analysis/test.html